$$\therefore s=-\tfrac{1}{2}\left[\frac{a^{\beta}\,(a-x)^{1-\beta}}{1-\beta}+\frac{a^{-\beta}\,(a-x)^{1+\beta}}{1+\beta}\right]+C_3.\quad \text{For } x=0,\ s=0.$$

$$\therefore C_3=\frac{a}{1-\beta^2}.\quad \therefore s=\frac{a}{1-\beta^2}-\tfrac{1}{2}\left[\frac{a^{\beta}\,(a-x)^{1-\beta}}{1-\beta}+\frac{a^{-\beta}\,(a-x)^{1+\beta}}{1+\beta}\right].$$

Since for $x=a$, $y=b$, we have $b=\dfrac{a\beta}{1-\beta^2}$; or, substituting $a=100$, $b=300$, we have $\beta^2+\frac{1}{3}\beta=1$; whence $\beta=\frac{1}{6}(\sqrt{37}-1)$ and length of curve between A and $C=\dfrac{a}{1-\beta^2}=50(\sqrt{37}+1)=354.135$.

$$y=\frac{na}{n^2-1}-\frac{na^{1/n}}{2(n-1)}(a-x)^{(n-1)/n}+\frac{n}{2(n+1)}\,\frac{1}{a^n}(a-x)^{(n-1)/n}.$$

315. Proposed by C. N. SCHMALL, New York City.

If $y=f(x)$, show by Taylor's Theorem that

$$f\left(\frac{x}{1+x}\right)=y-\frac{x^2}{1+x}\cdot\frac{dy}{dx}+\frac{x^4}{2\,(1+x)^2}\cdot\frac{d^2y}{dx^2}-\frac{x^6}{2.3.\,(1+x)^3}\cdot\frac{d^3y}{dx^3}+\dots\text{ etc.}$$

Solution by the PROPOSER.

Put $\dfrac{x}{1+x}=x+h$, then $f\left(\dfrac{x}{1+x}\right)=f(x+h)$,

and also, $h=\dfrac{x}{1+x}-x=-\dfrac{x^2}{1+x}$.

$\therefore h^2=\dfrac{x^4}{(1+x)^2}$, $h^3=-\dfrac{x^6}{(1+x)^3}$, and so on.

Substituting these values of the powers of h in Taylor's series, we have the required result; *i. e.*, from

$$f(x+h)=f(x)+hf'(x)+\frac{h^2}{2!}f''(x)+\frac{h^3}{3!}f'''(x)+\text{etc.},$$

we get, by direct substitution,

$$f\left(\frac{x}{1+x}\right)=y-\frac{x^2}{1+x}\,\frac{dy}{dx}+\frac{x^4}{2\,(1+x)^2}\,\frac{d^2y}{dx^2}-\frac{x^6}{2.3(1+x)^3}\,\frac{d^3y}{dx^3}+\text{etc.}$$

316. Proposed by C. N. SCHMALL, New York City.

$$\int_0^\infty \frac{\cos ax}{1+x^2}dx=\tfrac{1}{2}\pi\, e^{-a}=\int_0^\infty \frac{x\sin ax}{1+x^2}dx.$$

(From Bromwich, *Theory of Infinite Series*, p. 442, ex. 5, and also from Carslaw, *Fourier's Series*, p. 113, ex. 12.) Prove this by any method.